HABILITATION THESIS REVIEWER'S REPORT

Masaryk University	
Applicant	Jörgen Linus Wulff
Habilitation thesis	Integrable deformations of strings
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The habilitation thesis of Dr. Wulff is devoted to integrable structures in the so-called gauge-string correspondence that constitutes a broad and rapidly developing area of modern theoretical physics. The importance of this research is sustained by the fact that until recently there were no available theoretical tools to compute the relevant quantities in gauge theories in the regime when the coupling between their fundamental constituents becomes strong. Since the gauge theories describe the physics of elementary particles, understanding their behaviour at strong coupling represents an outstanding problem. A recent world-wide remarkable progress in this direction was due to the principle discovery that some of these gauge theories may be alternatively viewed as string theories, that is the theories of extended one-dimensional objects governed by well-defined dynamical principles. Moreover, in some cases these string theories appear to be exactly solvable (integrable) which enables one, through the gauge-string correspondence, to compute the relevant gauge theory quantities at strong coupling.

Our current experience with the gauge-string correspondence is largely based on the presence of a large amount of symmetry (conformal and supersymmetry) which raises an important question on whether this correspondence continues to hold if some of the symmetries get broken, for instance, by means of deformations. On the string theory side, a tempting possibility is offered by integrable deformations of strings propagating in the AdS spaces and this is precisely where the work of Dr. Wulff makes an original and essential contribution.

The thesis summarizes the research carried out by Dr. Wulff on integrable deformations of sigma models, including those relevant for the gauge-string correspondence. It is based on nine papers published in peer-reviewed journals and also contains an extended introductory part that highlights some important results by the author and provides the necessary background material.

After an introduction describing the structure of the thesis, in chapter 2 the so-called Yang-Baxter deformations of the prominent sigma model, the Principle Chiral Model (PCM) are introduced and discussed. While the PCM is not yet the string sigma model, it provides a viable example to study integrable deformations. Dr. Wulff also introduces another important tool, namely, non-abelian T-duality. This duality amounts to a certain non-local change of sigma model variables to get a dual model which has less amount of manifest global symmetry but still equivalent to the original model, at least at the classical level. Apparently non-abelian T-duality is a canonical transformation and, therefore, preserves integrability. As the author shows, application of non-abelian T-duality to the PCM leads to a particular class of homogeneous Yang-Baxter deformations.

In chapter 3 the bosonic string sigma model is introduced. In addition to the background metric it includes an anti-symmetric tensor field and a scalar. An essential new feature of this model is a requirement of its Weyl invariance which results into the generalized Einstein's equations that must be satisfied by the background fields.The fulfillment of the Weyl invariance conditions for the corresponding deformed model appears a highly non-trivial task and to solve it the author employs a reformulation of the string sigma model in terms of the Double Field Theory formalism that supports an extended O(D,D) symmetry. Further, Dr. Wulff applies this approach to study the conditions of Weyl invariance at the one-loop level. The main result is that at one loop the deformed background satisfies the generalized Einstein's equations provided a certain algebraic condition is met. This algebraic condition, called unimodularity condition, selects a class of classical R-matrices that govern the Yang-Baxter deformations compatible with Weyl invariance.

In chapter 4 the question of Weyl invariance in the deformed theory is investigated at the two-loop level. This is the case when Einstein's equations for the background fields receive the first alpha' correction. Dr. Wulff explains how to incorporate this correction in the Double Field Theory formalism. He further shows that the deformation itself must be corrected to satisfy the conditions of Weyl invariance at the two-loop order.

Concerning the Conclusions, one sentence caught my attention. There Dr. Wulff writes ``When (G+B)_{mn} of the undeformed background is invertible one finds no solution in the inhomogeneous case (although our analysis for the YB model with WZ-term is not quite complete)." As a possible question for the oral defence, I would ask what is an obstacle here and which results are missing in order to complete the analysis of the non-degenerate case.

Concluding my report, I find the thesis very clearly written with the summary and explanation of the most important and original scientific results of Dr. Wulff. The habilitation thesis entitled "Integrable deformations of strings" by Jörgen Linus Wulff fulfills requirements expected of a habilitation thesis in the field of Theoretical Physics and Astrophysics.

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